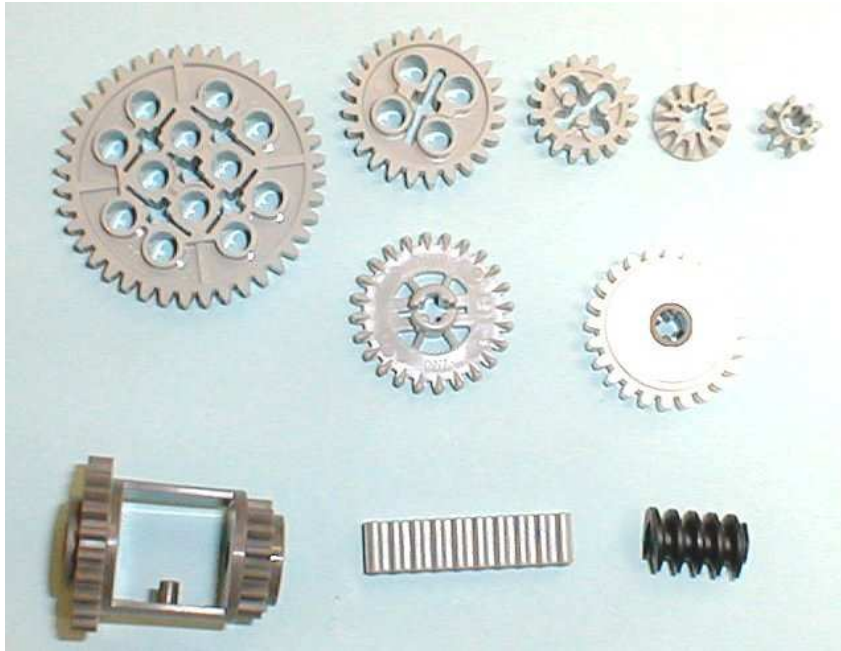


# Gear Measurement

## Topics

- Introduction
- Gear schematics
- Types of gears
- Measuring gears

# What is a gear ?



A gear is a wheel with teeth that mesh together with other gears.

Gears change the :

- speed
- torque (rot. force)
- direction of rotating axles.

## What is a gear ? (cont'd)

- Purpose : to transmit rotary motion and force.
- Gears are used in groups of two or more.
- A group of gears is called a gear train.
- The gears in a train are arranged so that their teeth closely interlock or mesh.
- The teeth on meshing gears are the same size so that they are of equal strength. The spacing of the teeth is the same on each gear.

# Gear schematic

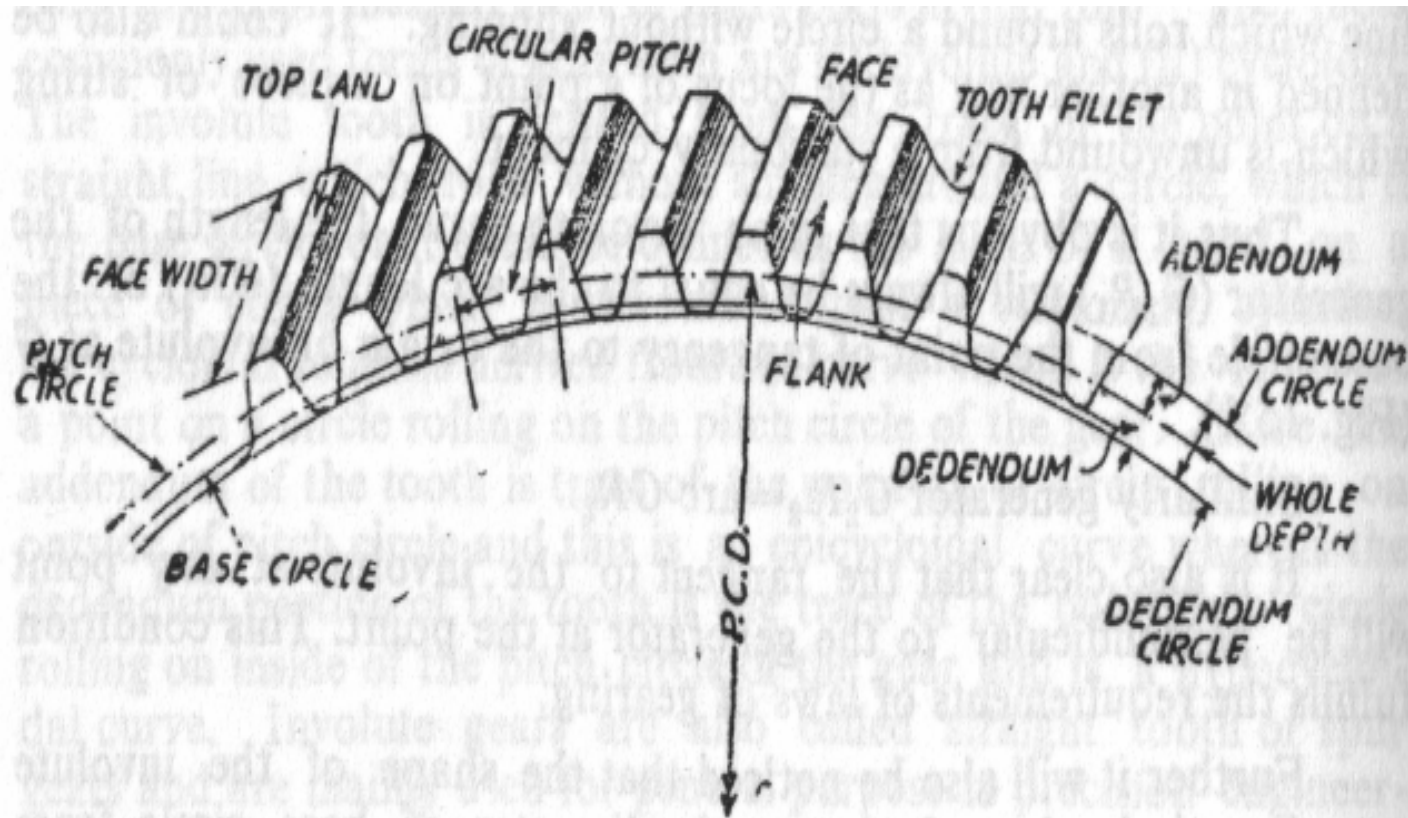


Figure : Gear schematic

## Gear schematic (cont'd)

- **Base circle** : The circle from which involute form is generated. Only the base circle on a gear is fixed and unalterable.
- **Pitch circle** : It is an imaginary circle most useful in calculations. An infinite number of pitch circles can be chosen, each associated its own pressure angle.
- **Pitch Circle Diameter (PCD)** : Diameter ( $D$ ) of a circle which by pure rolling action would produce the same motion as the toothed gear wheel. This is the most important diameter in gears.
- **Module** : Defined as the length of the pitch circle dia. per tooth ☐

$$\text{module } (m) = \frac{D}{N}; D = \text{PCD}, N = \text{number of teeth}$$

- **Circular pitch (CP)** : The arc distance measured around the pitch circle from the flank of one tooth to a similar flank in the next tooth.

$$CP = \frac{nD}{N} = \pi m$$

- **Addendum** : The radial distance between the pitch circle and tip of the tooth (outside diameters).

## GEAR SCHEMATIC (CONT'D)

- **Clearance** : Radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Standard value =  $0.157m$ .
- **Dedendum** : The radial distance from the pitch circle to the bottom of the tooth space :

$$Dedendum = Addendum + Clearance = m + 0.157m = 1.157m$$

- **Blank diameter** : Dia. Of the blank from which gear is cut  $\varnothing$  equal to PCD plus twice the addenda :

$$Blank\ diameter = PCD + 2m = mN + 2m = m(N + 2)$$

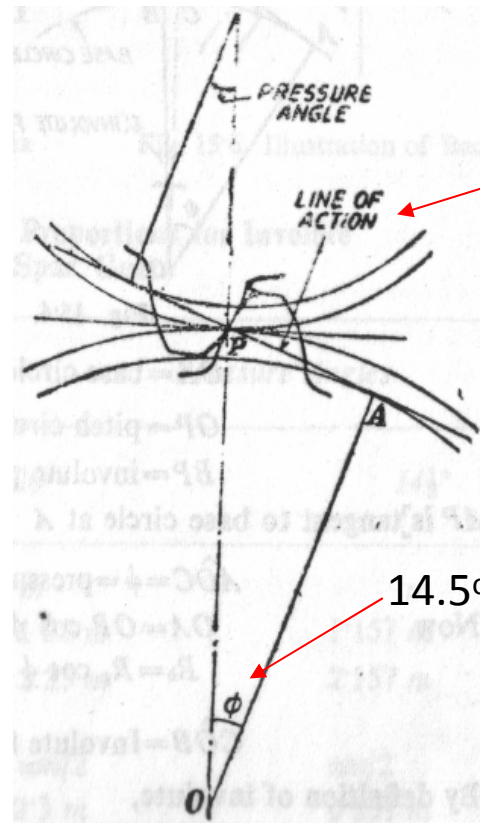
- **Tooth thickness** : Arc distance measured along the pitch circle from its intercept with one flank to its intercept with other flank of the same tooth :

$$Tooth\ thickness = 0.5CP = (\pi m) / 2$$

- **Face of tooth** : Part of the tooth surface which is above the pitch surface.
- **Flank of the tooth** : Part of the tooth surface which is lying below the pitch surface.

## GEAR SCHEMATIC (CONT'D)

- **Line of Action and Pressure Angle** : The teeth of a pair of gears in mesh, contact each other along the common tangent to their base circles ☐



The load and pressure between the gears transmitted along this line

14.5° or 20°

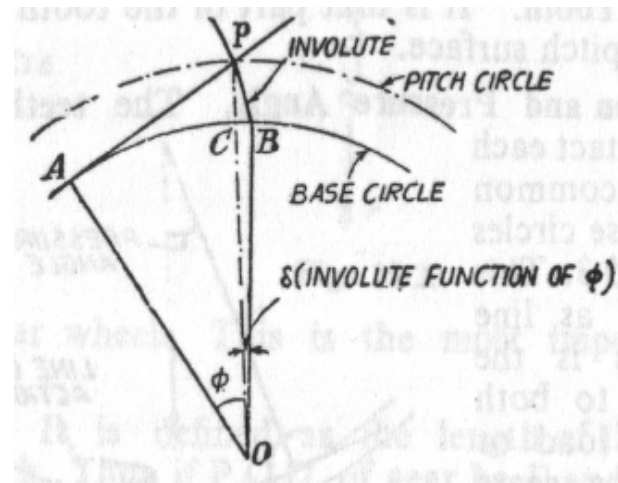
## GEAR SCHEMATIC (CONT'D)

- **Base Pitch** : Arc distance measured around the base circle from the origin of the involute on one tooth to the origin of a similar involute on the next tooth.

$$\begin{aligned} \text{Base Pitch} &= \frac{\text{Base circumference}}{\text{no. of teeth}} = \frac{\pi \times \text{dia. of base circle}}{N} = \frac{\pi D \cos \phi}{N} \\ &= \pi m \cos \phi \end{aligned}$$

- **Involute functions** : Found from the fundamental principle of the involute, that is the locus of the end of a thread unwound from the base circle.

*Involute function  $\delta = \tan \phi - \phi$*



## GEAR SCHEMATIC (CONT'D)

- **Helix angle** : Acute angle between the tangent to the helix and axes of the cylinder on which teeth are cut.
- **Lead angle** : Acute angle between the tangent to the helix and a plane perpendicular to the axis of cylinder.
- **Back lash** : The distance through which a gear can be rotated to bring its non-working flank incontact with the teeth of mating gear

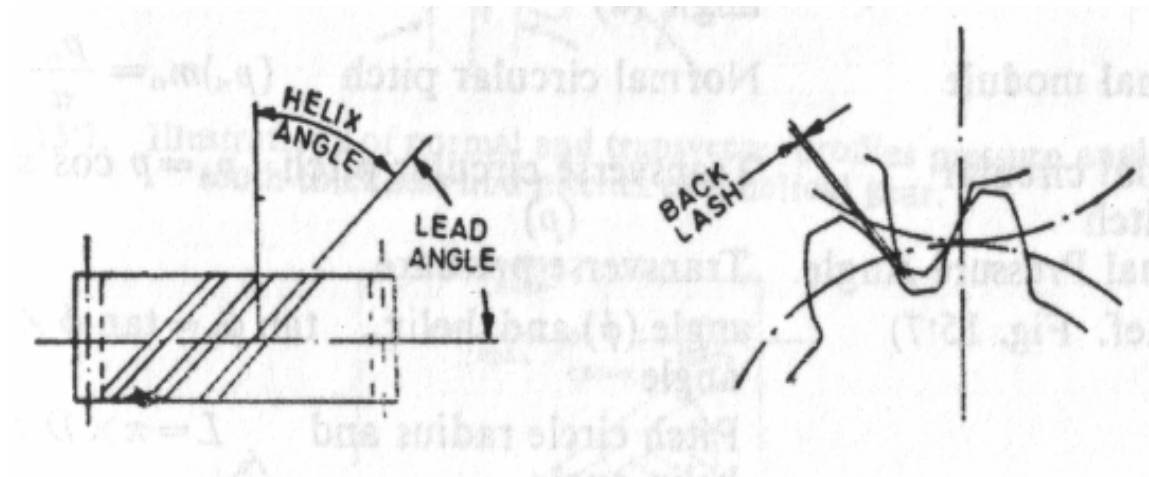


Figure : Helix and lead angle, and backlash

# Finding various elements of gears

| To find | Having | Formula |
|---------|--------|---------|
|---------|--------|---------|

## Spur Gears :

|                                |                                                 |                     |
|--------------------------------|-------------------------------------------------|---------------------|
| Module ( $m$ )                 | No. of teeth ( $N$ ) and pitch diameter ( $D$ ) | $m = \frac{D}{N}$   |
| Module                         | Circular pitch ( $p$ )                          | $m = \frac{p}{\pi}$ |
| Outside diameter. ( $D_o$ )    | Pitch diameter and Module                       | $D_o = D + 2m$      |
| Base circle diameter ( $D_b$ ) | Pitch diameter & pressure angle                 | $D_b = D \cos \phi$ |

## Helical Gears :

|                                                  |                                                                   |                                              |
|--------------------------------------------------|-------------------------------------------------------------------|----------------------------------------------|
|                                                  | (Refer Fig. 5)                                                    |                                              |
| Normal module ( $m_n$ )                          | No. of teeth, pitch diameter ( $D$ ) and helix angle ( $\alpha$ ) | $m_n = \frac{D}{N} \cos \alpha$              |
| Normal module                                    | Normal circular pitch ( $p_n$ )                                   | $(p_n)m_n = \frac{p_n}{\pi}$                 |
| Normal circular pitch ( $p$ )                    | Transverse circular pitch ( $p$ )                                 | $p_n = p \cos \alpha$                        |
| Normal Pressure Angle. $\phi_n$ (Ref. Fig. 15.7) | Transverse pressure angle ( $\phi$ ) and helix angle              | $\tan \phi_n = \tan \phi \times \cos \alpha$ |
| Lead                                             | Pitch circle radius and helix angle.                              | $L = \pi \times D \cot \alpha$               |
| Transverse teeth thickness ( $t$ )               | Normal tooth thickness ( $t_n$ ), and helix angle.                | $t = \frac{t_n}{\cos \alpha}$                |

## FINDING VARIOUS ELEMENTS OF GEARS (CONT'D)

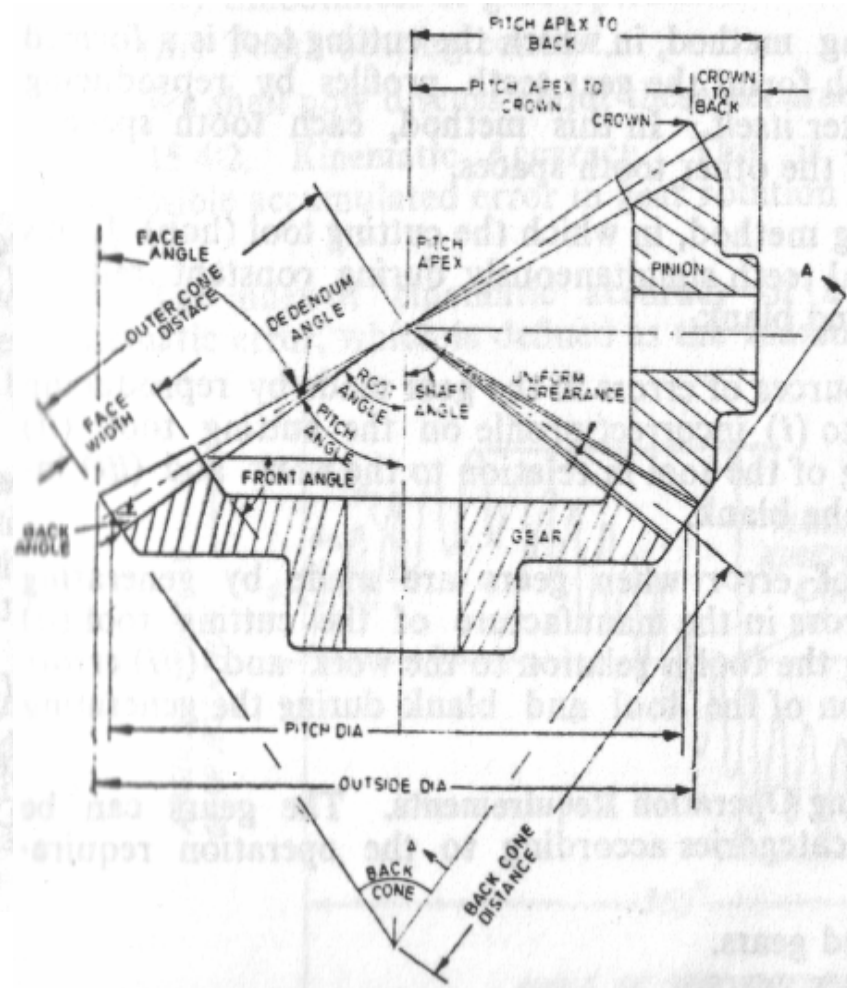
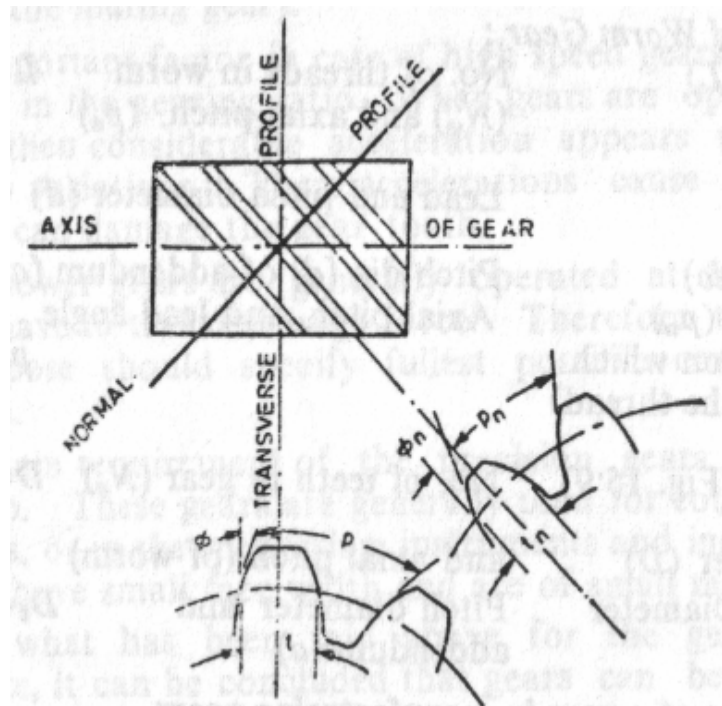
| To find | Having | Formula |
|---------|--------|---------|
|---------|--------|---------|

Bevel Gears :

Fig. 15·8)

|                                            |                                                             |                                                             |
|--------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|
| Pitch dia of pinion<br>( $D_p$ )           | No. of pinion teeth<br>( $n$ ) and module,                  | $D_g = n.m.$                                                |
| Pitch dia of gear<br>( $D_g$ )             | No. of gear teeth ( $N$ )<br>and module.                    | $D_g = N m.$                                                |
| Pitch angle of Pinion ( $\beta_p$ )        | No. of pinion teeth and<br>No. of gear teeth                | $\beta\phi = \tan^{-1} \frac{n}{N}$                         |
| Pitch angle of gear ( $\beta_g$ )          | Pitch Angle of pinion                                       | $\beta_g = (90 - \beta_p)^\circ$                            |
| Outer Cone distance<br>(for pinion & gear) | Gear pitch diameter and<br>pitch angle of gear.             | $A_o = \frac{D_g}{2 \sin p_g}$                              |
| Circular Pitch ( $p$ )                     | $m$                                                         | $p = m \times \pi$                                          |
| Dedendum angle<br>or pinion or gear        | Dedendum of<br>pinion or gear<br>and outer Cone<br>distance | Ded. angle<br>$= \frac{\text{Ded.}}{A_o} (p \text{ or } g)$ |

## FINDING VARIOUS ELEMENTS OF GEARS (CONT'D)



# FINDING VARIOUS ELEMENTS OF GEARS

## Bevel Gears :

|                                                                             |                                                             |                                  |
|-----------------------------------------------------------------------------|-------------------------------------------------------------|----------------------------------|
| Worm Lead ( $L$ )                                                           | No. of threads in worm ( $N_w$ ) and axial pitch. ( $p_x$ ) | $L = N_w \times P_x$             |
| Lead angle $\psi$                                                           | Lead and pitch diameter ( $d$ )                             | $\tan \psi = \frac{L}{\pi d}$    |
| Outside dia ( $d_o$ )                                                       | Pitch dia ( $d$ ) of addendum ( $a$ )                       | $d_o = d + 2a$                   |
| Normal pitch ( $p_n$ )<br>(i.e. in direction which is normal to the thread) | Axial pitch, and lead angle                                 | $p_n = p_x \times \cos \psi$     |
| Worm Gear (Fig. 15.9)                                                       | No. of teeth in gear ( $N_g$ )                              | $D = \frac{N_g \times p_x}{\pi}$ |
| Pitch Diameter ( $D$ )                                                      | and axial pitch (of worm)                                   |                                  |
| Throat ( $D_t$ ) Diameter                                                   | Pitch diameter and addendum ( $a$ )                         | $D_t = D + 2 \times a$           |

# Different types of gears

Spur gears



Helical gears



Worm gears



Bevel gears



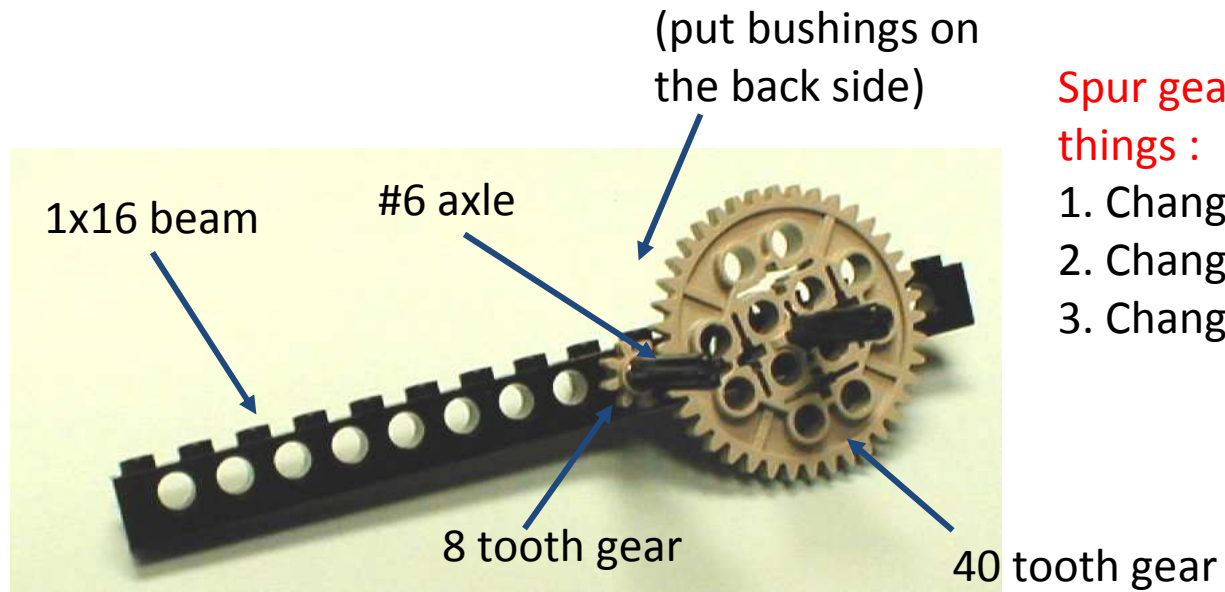
Straight gears



# Spur Gears

Most “common” type of gear, a wheel with teeth.

Build the following...



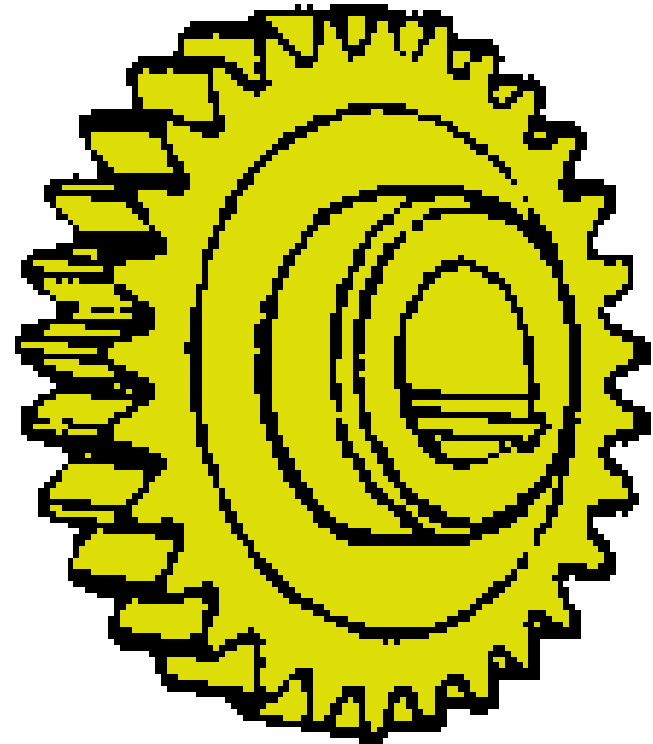
Spur gears do three things :

1. Change rot. speed
2. Change torque
3. Change direction

Make sure there isn't too much friction between the gears and the beam. The gears should spin easily.

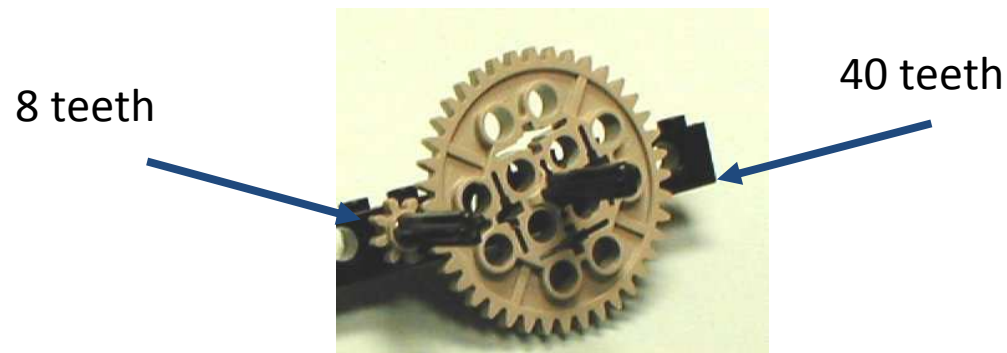
## Spur Gears (cont'd)

- **Advantages** : Spur gears are easy to find, inexpensive, and efficient.
- **Disadvantages**: Spur Gears generally cannot be used when a direction change between the two shafts is required.



# Gear Ratios

The gear ratio is the ratio of the number of teeth on one gear to the number of teeth on the other gear.



Gear ratio = 40 to 8 or, simplifying, 5 to 1.

1. That means it takes 5 revolutions of the smaller gear to get 1 revolution of the larger gear. Try it!

## Gear Ratios (cont'd)

2. The gear ratio tells the change in speed and torque of the rotating axles.
3. If it takes 5 turns of the 8 tooth gear for every 1 turn of the 40 tooth gear, that means the 40 tooth gear will rotate 5 times slower than the 8 tooth gear.
4. BUT, it also means the 40 tooth gear's axle has 5 times the torque (rotational force) as the 8 tooth gear's axle.

# Helical Gears

**Helical Gears** are similar to the **Spur gears** except that the teeth are at an angle to the shaft, rather than parallel to it as in a Spur Gear. The resulting teeth are longer than the teeth on a spur gear of equivalent pitch diameter. The longer teeth cause helical gears to have the following differences from spur gears of the same size:

1. Tooth strength is greater because the teeth are longer.
2. Greater surface contact on the teeth allows a helical gear to carry more load than a spur gear.
3. The longer surface of contact reduces the efficiency of a helical gear relative to a spur gear.

## HELICAL GEARS (CONT'D)

**Advantages** : Helical Gears can be used on non parallel and even perpendicular shafts, and can carry higher loads than can Spur Gears.

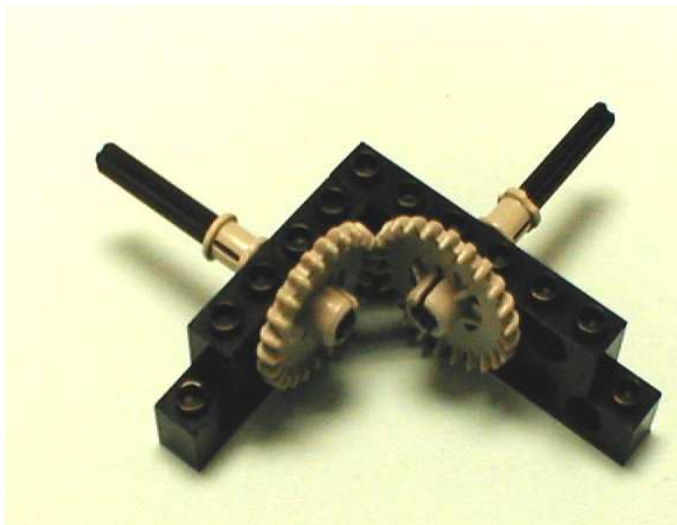
**Disadvantages** : Helical Gears have a major disadvantage that they are noisier and much more difficult to find. Helical Gears are also slightly less efficient than a Spur Gear of the same size.



# Bevel Gears

## Bevel gears

- are **spur gears that mesh at a 90 degree angle**. The gear ratio rules remain the same, but the axles are perpendicular to one another.

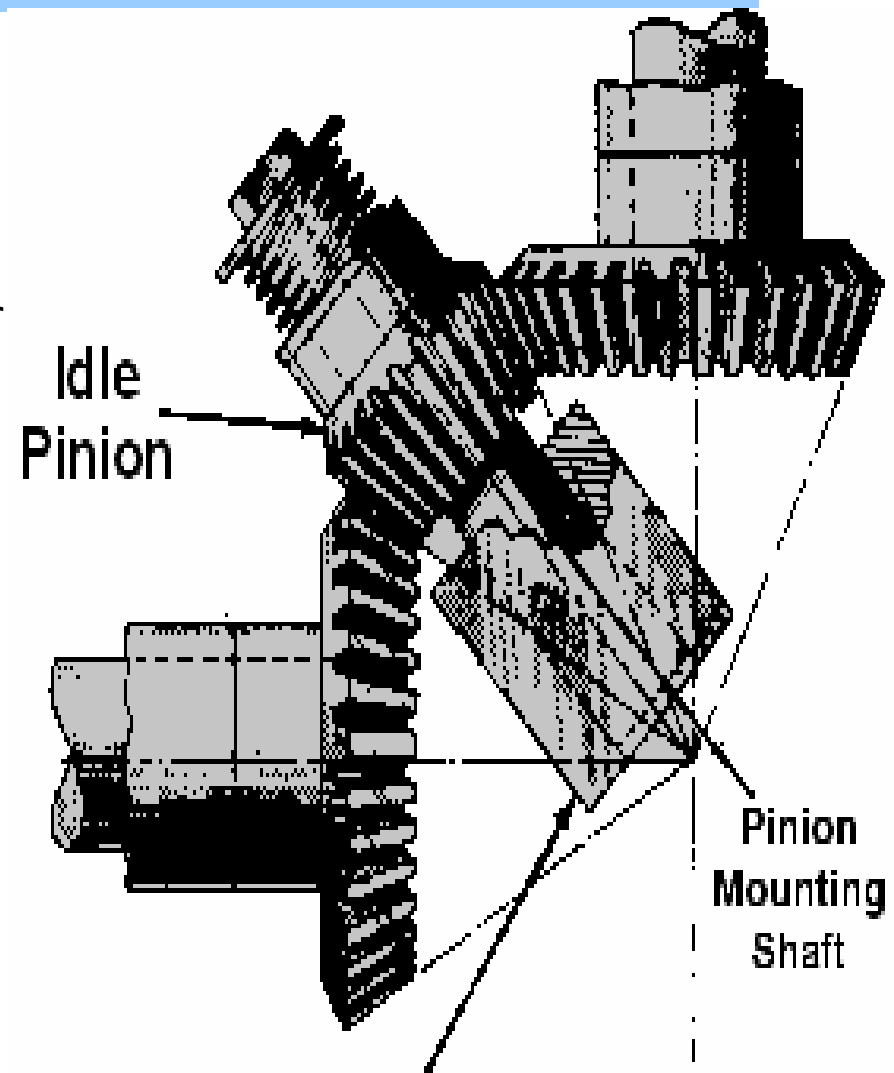


These 12 tooth bevel gears can only mesh with them-selves.

## Bevel Gears (cont'd)

**Advantages** : Excellent choice for intersecting shaft systems.

**Disadvantages** : Limited availability. Cannot be used for parallel shafts. Can become noisy at high speeds.



# Worm Gears

## Worm Gears :

1. Special gears that resemble screws, and can be used to drive **Spur Gears** or **Helical Gears**.
2. Like Helical Gears, allow two non-intersecting 'skew' shafts to mesh. Normally, the two shafts are at right angles to each other.
3. **Equivalent to a V-type screw thread.**
4. Normally used when a high gear ratio is desired, or again when the shafts are perpendicular to each other. One very important feature of Worm Gear Meshes that is often of use is their *irreversibility*

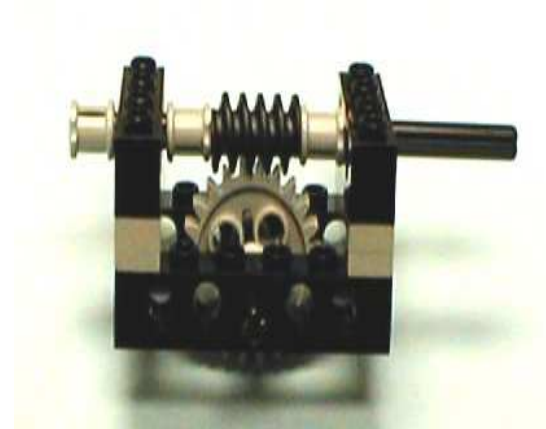


## Worm Gears (cont'd)

**Advantages** : Will tolerate large loads and high speed ratios. Meshes are self locking (which can be either an advantage or a disadvantage).

**Disadvantages** : Low efficiency. The Worm drives the drive gear primarily with slipping motion, thus there are high friction losses.

Worm gears are not back-driveable !



You can turn the worm gear's axle, but you can't turn the spur gear's axle.

This property is used as a locking mechanism.

# Straight Gear

**Racks** are **Straight Gears**  $\Rightarrow$  used to convert rotational motion to translational motion by means of a gear mesh. (They are in theory a gear with an infinite pitch diameter).

In theory, the torque and angular velocity of the Pinion Gear are related to the Force and the velocity of the rack by the radius of the pinion gear. Perhaps the most well-known application of a rack is the rack and pinion steering system used on many cars in the past.

**Advantages** : The only gearing component that converts rotational motion to translational motion. Efficiently transmits power. Generally offers better precision than other conversion methods.

**Disadvantages** : Limited usefulness. Difficult to find.



# Measurement and Testing of Gears

Two common forms of gears :

1. **Involute** derived from the trace of the point on a straight line, which rolls without slipping around the circle.
2. **Cycloidal** derived from the curve which is the locus of a point on a circle rolling on the pitch circle of the gear.

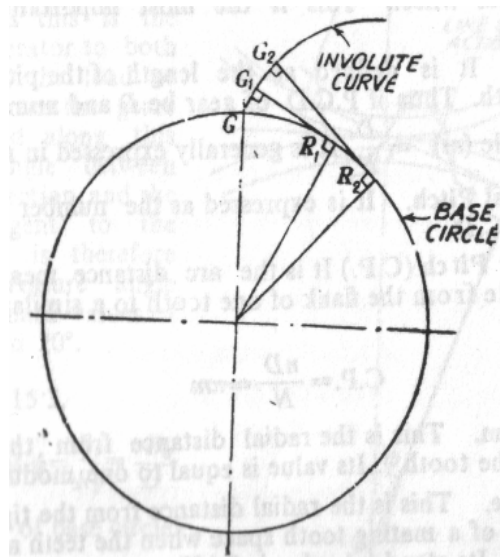


Figure : Involute curve

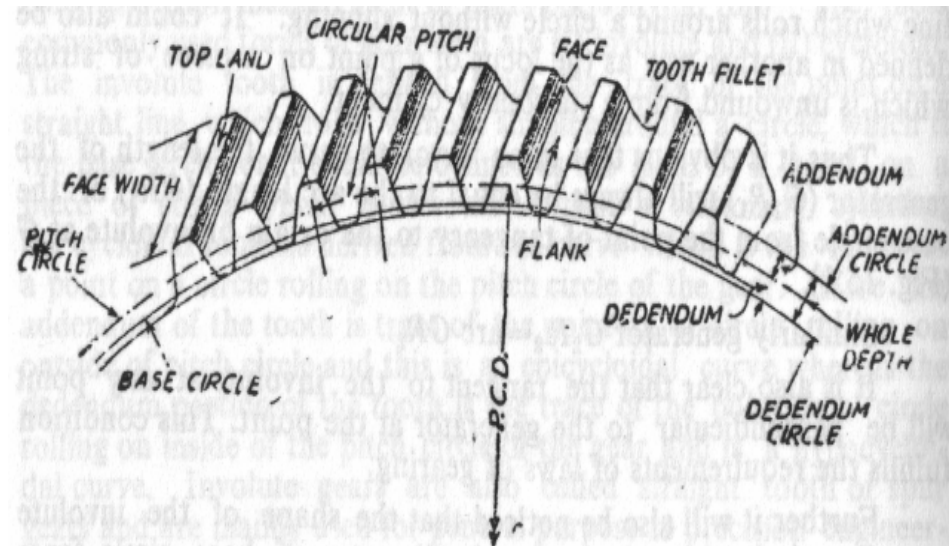


Figure : Schematic of involute gears

## MEASUREMENT AND TESTING OF GEARS (CONT'D)

Two methods in manufacturing gears :

1. Reproducing method ☐ the cutting tool is a formed **involute cutter**, which forms the gear teeth profiles by reproducing the shape of the cutter itself ☐ **each tooth space is cut independently of the other tooth spaces.**
  - Gear errors due to : i) **incorrect profile of the cutting tool**, ii) **incorrect positioning of the tool wrt the work**, iii) **incorrect indexing of the blank.**
2. Generating method ☐ the cutting tool (hob) **forms the profiles of several teeth simultaneously during constant relative motion of the tool and blank.**
  - Gears errors due to : i) **errors in the manufacture of the cutting tool**, ii) **errors in positioning the tool wrt the work**, iii) **error in the relative motion during the generating.**

# Measurement and Testing of Gears (cont'd)

Gear accuracies graded based on :

1. Kinematic accuracy ? Maximum kinematic error defined as : a) maximum angular error per revolution of a gear when meshes with master gear, b) maximum accumulated error in circular pitch.

- Errors due to : a) errors in positioning the tool wrt the tool (when the gear is being cut), b) kinematic errors of the gear cutting machine.

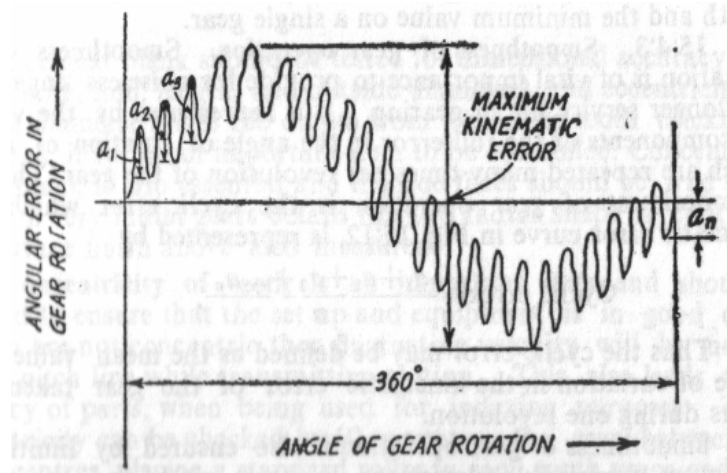


Figure : kinematic error curve

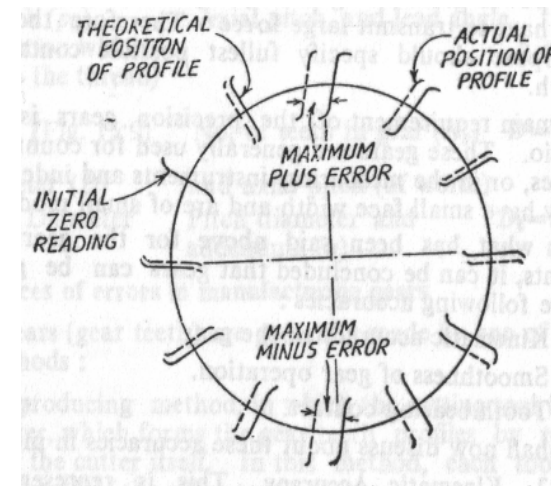


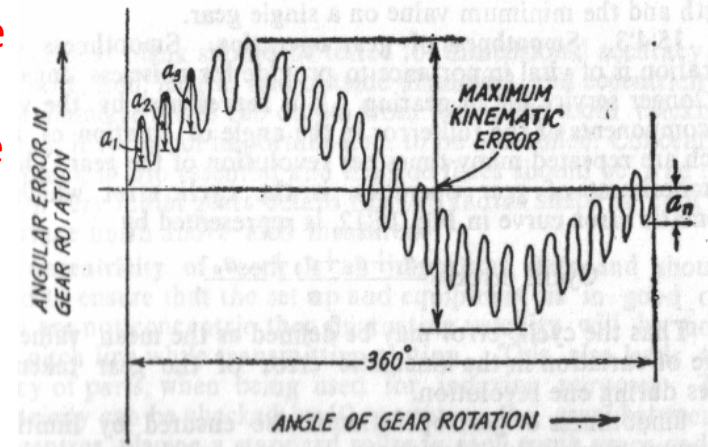
Figure : circular pitch error

## MEASUREMENT AND TESTING OF GEARS (CONT'D)

### 2. Smoothness of gear operation ?

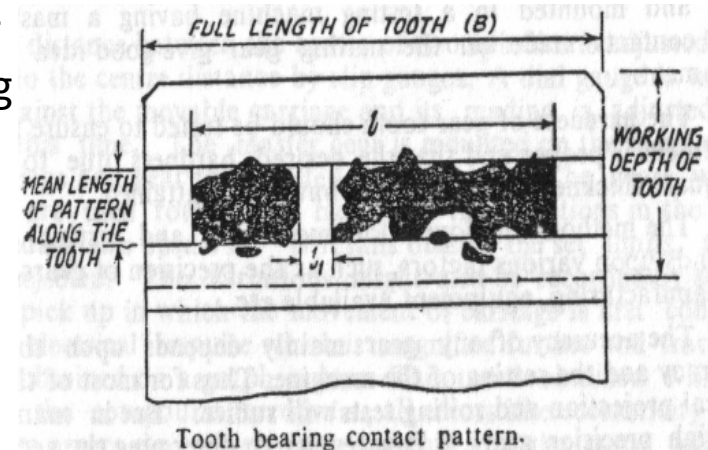
- mean value of variation in the kinematic error of the gear taken for all cycles during one revolution ? cyclic error.

Cyclic error =  $\frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$   
to provide noiseless engagement and longer service life.



### 3. Tooth bearing contact ? gear contact pattern ?

part of the surface on a gear tooth profile showing traces of bearing contact with the teeth of the mating gear after running together with the driven gear braked slightly.



$$\text{Bearing contact} = \frac{l - l_1}{B} \times 100\%$$

## MEASUREMENT AND TESTING OF GEARS (CONT'D)

### Gear measurement :

#### 1. Teeth concentricity ☐

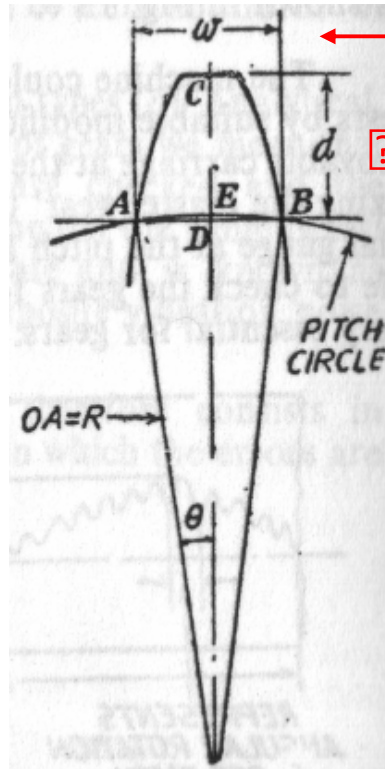
- Checked by : i) mounting the gear between the bench center, placing a standard roller in each tooth space and using dial indicator, ii) using projector ☐ matching the tooth image with a line on the screen, iii) etc.
- If teeth are not concentric ☐ fluctuating velocity !

#### 2. Good alignment of each tooth ☐

- 1<sup>st</sup> method = checked by placing a standard roller in the tooth space and checking for parallelism off a surface plate.
- 2<sup>nd</sup> method = teeth on one gear are lightly marked with prussion blue and mounted in a testing machine having a maaster gear ☐ see contact results !

#### 3. Gear hardness.

# MEASUREMENT OF TOOTH THICKNESS



$$w = Nm \sin\left(\frac{90}{N}\right)$$

For Helical gears ?

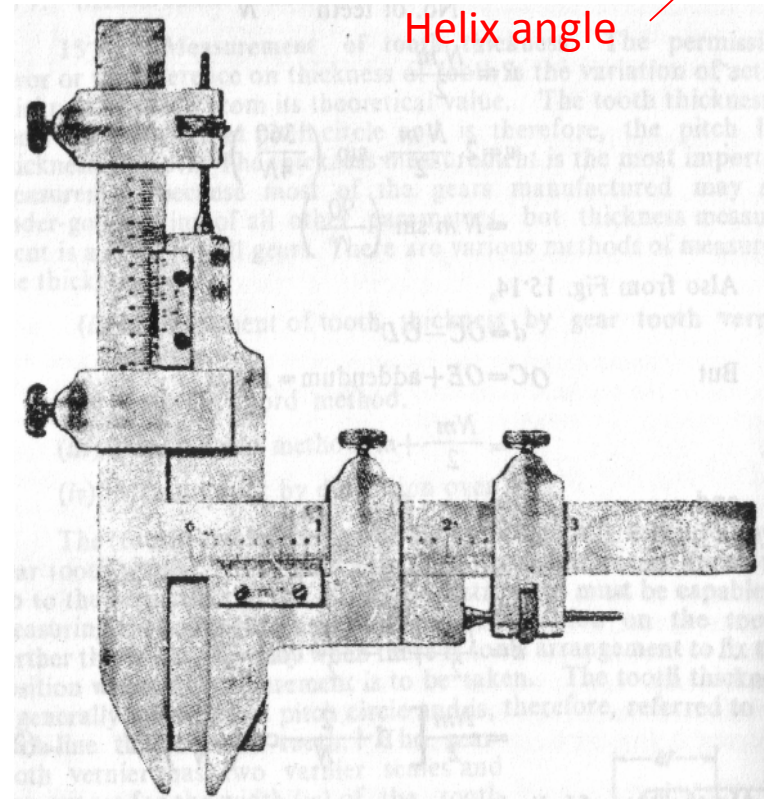
$$d = \frac{Nm}{2} \left[ 1 + \frac{2}{N} - \cos\left(\frac{90}{N}\right) \right]$$

w = chordal thickness  
d = chordal addendum

$$w = \frac{Nm_n}{\cos^3 \alpha} \sin\left(\frac{90}{N} \cos^2 \alpha\right)$$

$$d = \frac{Nm_n}{\cos^3 \alpha} \left[ 1 + \frac{2 \cos^3 \alpha}{N} - \cos\left(\frac{90}{N} \cos^3 \alpha\right) \right]$$

Helix angle



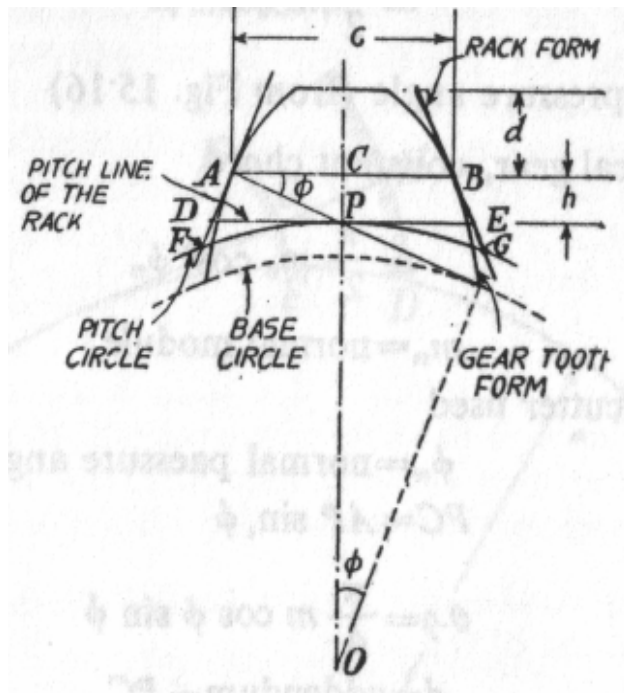
## 1. Gear Tooth Vernier Calliper ?

- Have 2 scales for width (w) and depth (d)

## MEASUREMENT OF TOOTH THICKNESS (CONT'D)

### 2. Constant chord method ☐

- Efficient for measuring a large number of gears, each having different number of teeth but the same module ( $m$ ).



AB = Constant chord

$$c = AB = 2AC$$

$$= \frac{\pi}{2} m \cos^2 \phi \rightarrow \phi = \text{pressure angle}$$

$$d = \text{addendum} - PC = m \left[ 1 - \frac{\pi}{4} \cos \phi \sin \phi \right]$$

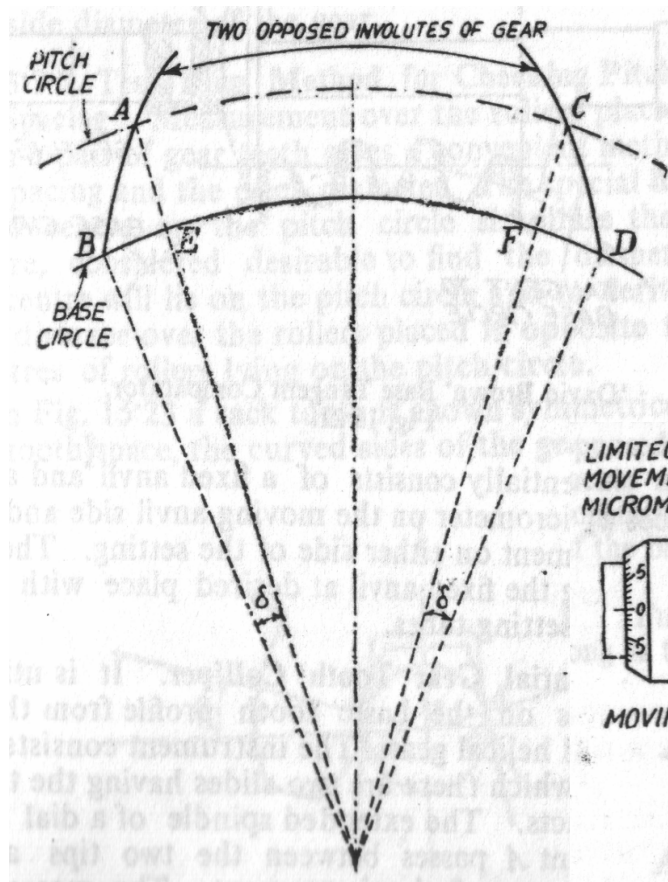
For Helical gears ☐

$$c = \frac{\pi}{2} m_n \cos^2 \phi_n$$

$$d = m_n \left[ 1 - \frac{\pi}{4} \cos \phi_n \sin \phi_n \right]$$

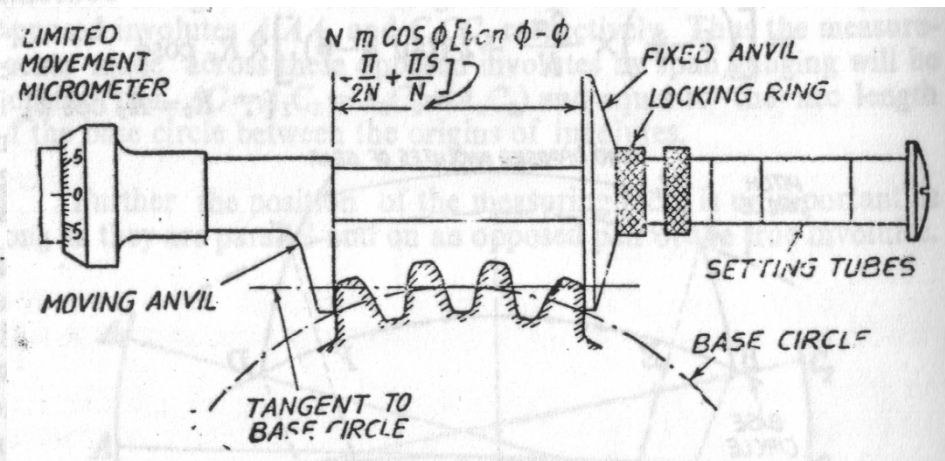
## MEASUREMENT OF TOOTH THICKNESS (CONT'D)

### 3. Base Tangent Method (David Brown tangent comparator) ☐.



$$BD = Nm \cos \phi \left[ \tan \phi - \phi - \frac{\pi}{2N} + \frac{\pi S}{N} \right]$$

$AC$  corresponds to  $S$  number of teeth



## MEASUREMENT OF TOOTH THICKNESS (CONT'D)

### 4. Dimension over the pins ☐.

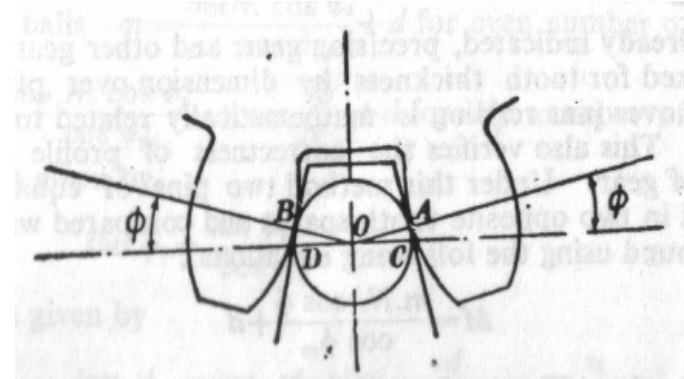
$$\text{Roller diameter} = \frac{\pi}{2} m \cos \phi$$

$$\text{Gauging dia. over rollers} = m \left[ N + \frac{\pi}{2} \cos \phi \right]$$

Size over the wires:

$$M = \frac{mN \cos \phi}{\cos \phi_m} + d \rightarrow \text{even no. teeth}$$

$$M = \frac{mN \cos \phi}{\cos \phi_m} \cos \frac{90}{N} + d \rightarrow \text{odd no. teeth}$$

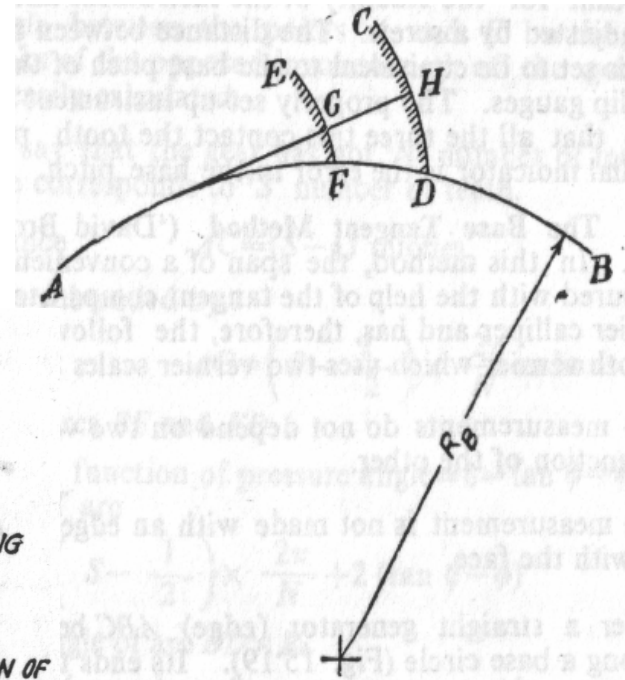
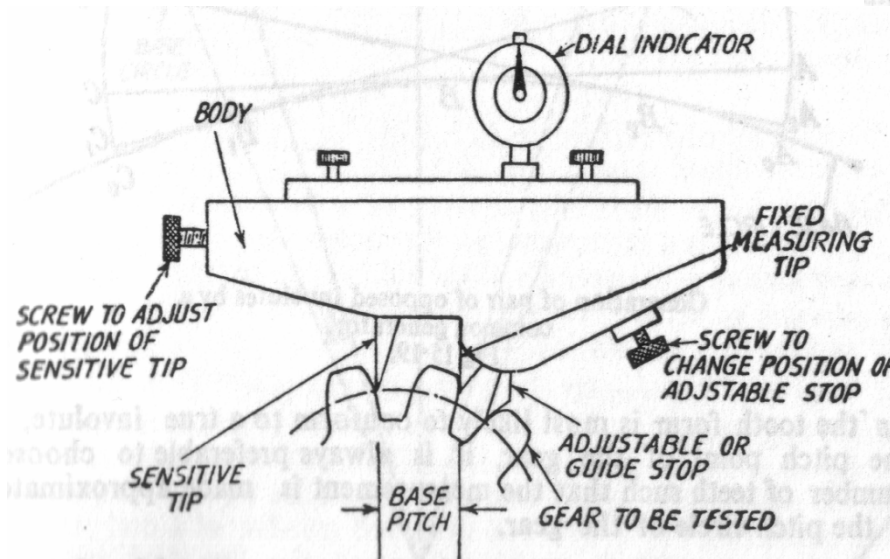


# MEASUREMENT OF BASE PITCH

Base Pitch is circular pitch of the teeth measured on the base circle.

$GH = FD$  from property of involute !

$$\begin{aligned} \text{Base Pitch} = FD &= \frac{2\pi}{N} \times \frac{PCD}{2} \times \cos \phi \\ &= \pi m \cos \phi \end{aligned}$$



# Measurement of Involute Shape of Gear

**Table**

| Accuracy class or grade of gear | Profile tolerance in microns |
|---------------------------------|------------------------------|
| 1                               | $2.0 + 0.06 \times k$        |
| 2                               | $2.5 + 0.10 \times k$        |
| 3                               | $3.0 + 0.16 \times k$        |
| 4                               | $4.0 + 0.25 \times k$        |
| 5                               | $5.0 + 0.40 \times k$        |
| 6                               | $6.3 + 0.63 \times k$        |
| 7                               | $8.0 + 1.0 \times k$         |
| 8                               | $10 + 1.6 \times k$          |
| 9                               | $16 + 2.5 \times k$          |
| 10                              | $25 + 4.0 \times k$          |
| 11                              | $40 + 6.3 \times k$          |
| 12                              | $63 + 10 \times k$           |

Here  $k$  is a tolerance factor and is given by  $k = m + 0.1\sqrt{D}$ .

Where  $m$  is module, and  $D$  is the pitch circle diameter in mm.

## MEASUREMENT OF INVOLUTE SHAPE OF GEAR (CONT'D)

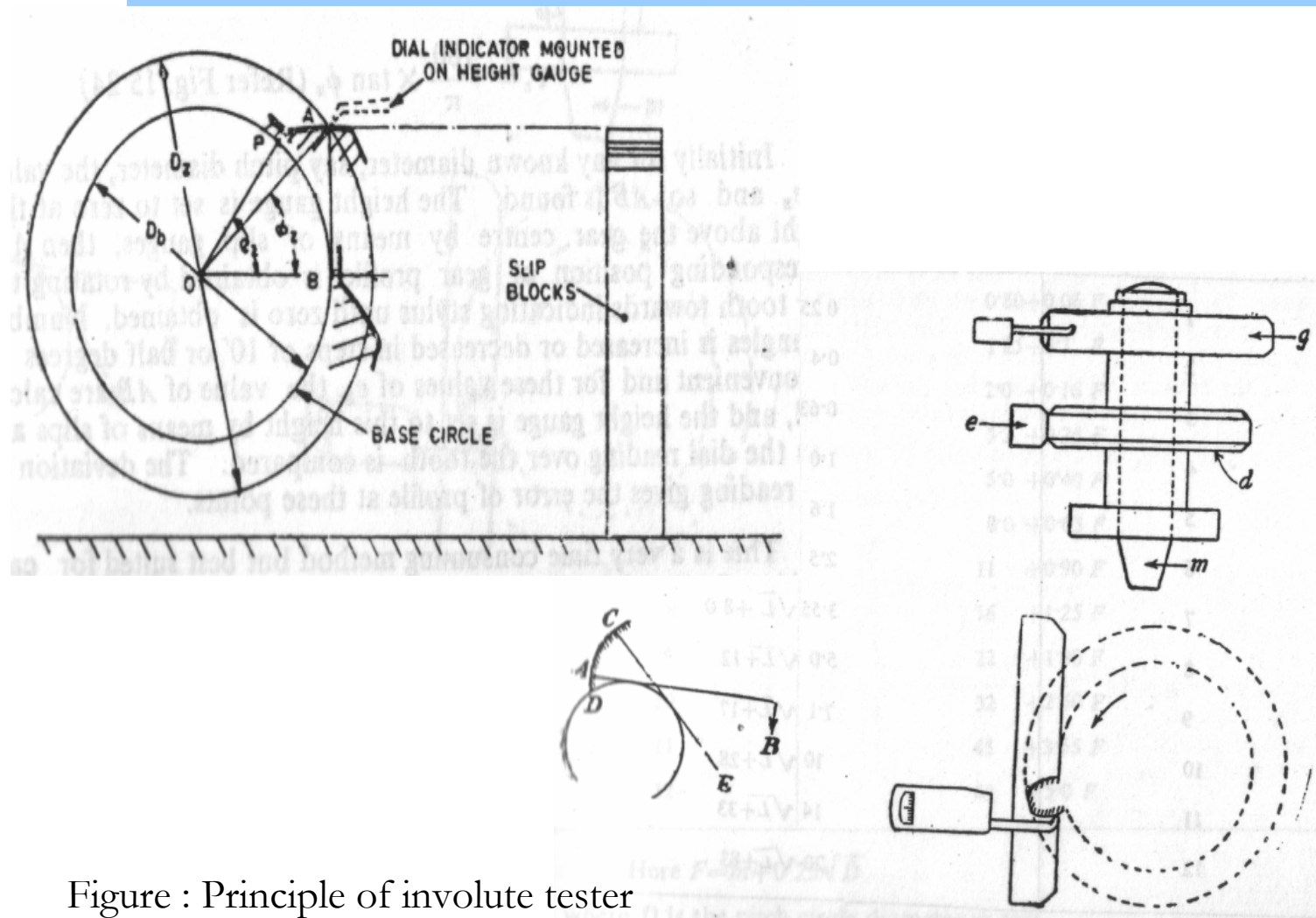


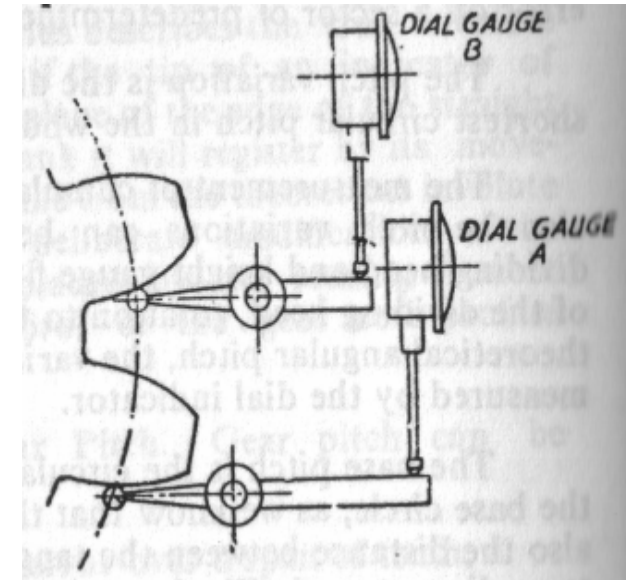
Figure : Principle of involute tester

# MEASUREMENT OF GEAR PITCH

**A. Tolerance over a sector of 'K' Pitch**

| Grade of Accuracy<br>Class of gear: | Permissible error in microns |
|-------------------------------------|------------------------------|
| 1                                   | $0.25\sqrt{L} + 0.6$         |
| 2                                   | $0.4\sqrt{L} + 1.0$          |
| 3                                   | $0.63\sqrt{L} + 1.6$         |
| 4                                   | $1.0\sqrt{L} + 2.5$          |
| 5                                   | $1.6\sqrt{L} + 4.0$          |
| 6                                   | $2.5\sqrt{L} + 6.0$          |
| 7                                   | $3.55\sqrt{L} + 8.0$         |
| 8                                   | $5.0\sqrt{L} + 12$           |
| 9                                   | $7.1\sqrt{L} + 17$           |
| 10                                  | $10\sqrt{L} + 28$            |
| 11                                  | $14\sqrt{L} + 33$            |
| 12                                  | $20\sqrt{L} + 83$            |

Here  $L = K\pi m^2$ , where  $K$  is the sector of pitches over which pitch error is to be checked.



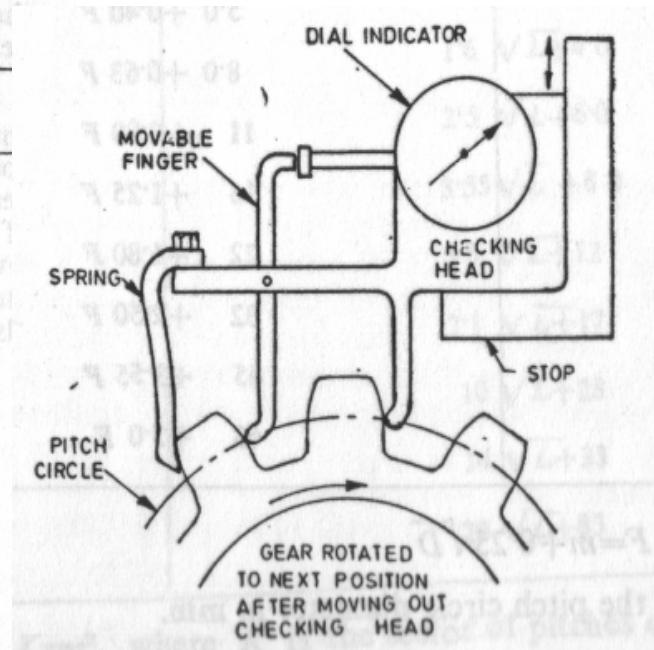
1. Index the gear through successive pitches ?
2. Any changes in the reading on dial gauge B indicate that pitch errors are present !

## MEASUREMENT OF GEAR PITCH (CONT'D)

**B. Tolerances on Pitch Variations**

| Class | Permissible error in microns |
|-------|------------------------------|
| 1     | $0.80 + 0.06 F$              |
| 2     | $1.25 + 0.1 F$               |
| 3     | $2.0 + 0.16 F$               |
| 4     | $3.2 + 0.25 F$               |
| 5     | $5.0 + 0.40 F$               |
| 6     | $8.0 + 0.63 F$               |
| 7     | $11 + 0.90 F$                |
| 8     | $16 + 1.25 F$                |
| 9     | $22 + 1.80 F$                |
| 10    | $32 + 2.50 F$                |
| 11    | $45 + 3.55 F$                |
| 12    | $63 + 5.0 F$                 |

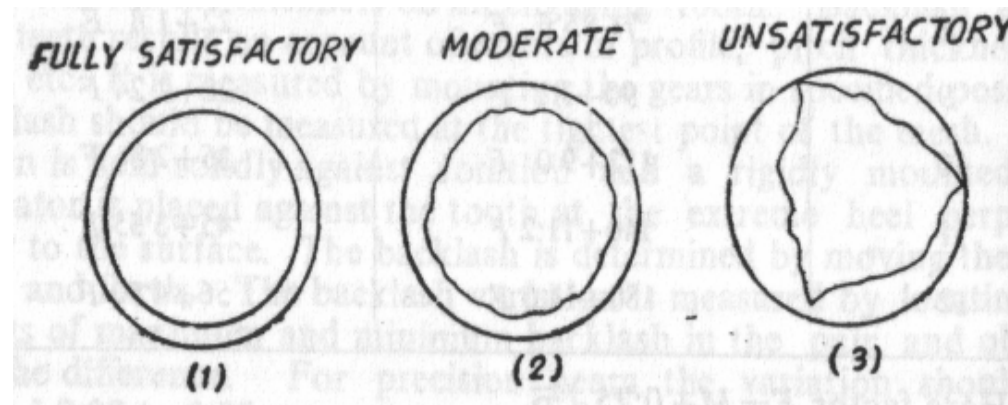
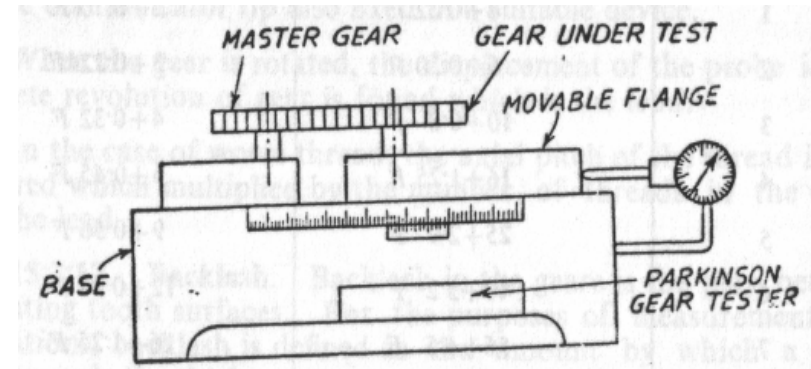
Here  $F = m + 0.25\sqrt{D}$   
 where  $D$  is the pitch circle diameter in mm.



## MEASUREMENT AND TESTING OF GEARS (CONT'D)

- **Parkinson Gear Tester**

- We just need to operating the device by putting the gear on the spindle then the needle will touch the gear then when the spindle starts to rotating the machine will calculate every thing that you need from a gear



## MEASUREMENT AND TESTING OF GEARS (CONT'D)

### ■ Parkinson Gear Tester

